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CONTROL OF THE SELF-OSCILLATION OF AMPLITUDE OF VIBRATION COMBUSTION IN A LIQUID-PROPELLANT ROCKET ENGINE BY SOLVING THE SYSTEM OF EQUATIONS THAT DESCRIBE THIS REGIME OF COMBUSTION

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Periodic solutions of the degenerate system of equations of nonstationary motion of a medium in a liquid-propellant rocket engine were obtained, with the aid of which the possibility of lowering the amplitude of the longitudinal self-oscillations of vibration combustion or their complete removal has been substantiated.

Keywords: *degenerate system, limiting cycle, control of the amplitude, vibration combustion.*

Introduction. Vibration combustion in a liquid-propellant rocket engine (LPRE) as in a dynamic system with discrete parameters is described by a nonlinear degenerate system of equations considered in a number of monographs, e.g., [1, 2], etc. This system has a periodic solution in the case of the phenomenological lag in the combustion of a fuel [3] and on increase in the pressure in a combustion chamber with the mass flow rate of fuel components [4], as a result of which the corresponding pressure head characteristic becomes saddle-like, as well as in the presence of a descending branch on the $h_n(G, T)$ characteristic of a propelling nozzle [2, 5] generally dependent on the mass flow rate of combustion products G and temperature T. The formation of the descending branch of the function $h_n(G, T)$ is due to the fact that the relationship between the fuel components changes with its total flow rate determined by impeller pumps with different pressure head characteristics. This is the reason for the nonstationarity of flow of combustion products issuing from a propelling nozzle [5]. Because of the fluctuations occurring in this case in the fuel flow rate, the relationship between the fuel components may periodically vary leading to the appearance of entropy waves in a combustion chamber. Thus, in the presence of the descending branch on the $h_n(G, T)$ characteristic self-oscillations are generated in a combustion chamber even in the absence of the phenomenological lag and of the ascending branch on the pressure head characteristic of a combustion chamber [6]. The increase in the amplitude of such oscillations can also be due to other mechanisms [7]. It should be noted that the possibility of exciting self-oscillations because of the formation of entropy waves was noted in [3], where it was stated that no theory resting on this mechanism had been proposed as yet, nor the reason for the formation of periodic entropy waves.

In experimental investigations of vibration combustion in the vertical chambers of the regenerative air heaters of blast furnaces it was found [8] that the same measures taken in different units to decrease the amplitudes of oscillations can cause a different and sometimes exactly opposite effect because of the specific features of the mechanisms that sustain the nonstationary motion of the burnt gas and air flow in them.

Statement of the Problem. The solutions of the system of equations of nonstationary motion of a medium in an LPRE are being found, and changes in these solutions on variation of various (bifurcation) parameters of the system are being determined with a view to controlling the self-oscillations of vibration combustion in it. Also determined is the influence of a constant phenomenological lag τ, of the acoustic parameters of the combustion chamber, and of the value of dF/dG, the ratio between the pressure head and fuel flow rate in the combustion chamber that determines the intensity of the scattering of pressure head on escape of combustion products from the propelling nozzle on the excitation of self-oscillations in an LPRE.

The character of the rearrangement of solutions is determined by the change in the limiting cycles or directly in the oscillations $p_n(t)$ of pressure ahead of the inlet into the nozzle. The possibilities of the reduction of the amplitude of oscillations, as well as of their complete suppression, have been justified.

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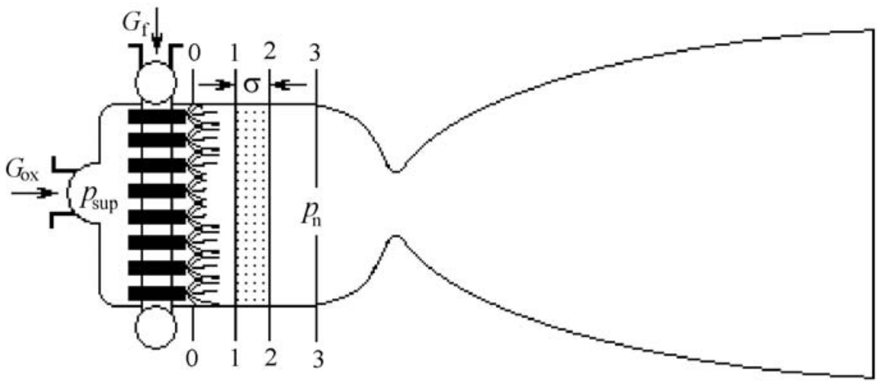

Fig. 1. Position of the sections in the LPRE combustion chamber that are used for construction of the equations of motion of combustion products.

Mathematical Description of the Dynamics of an LPRE in Vibration Combustion of a Fuel. The mass conservation equation [9] for the flow of a medium in the combustion chamber of an LPRE has the form

$$C_{\text{a,c.ch}} \frac{dp_{\tilde{n}}}{dt} = G\,(t-\tau) - G_{\text{n}}\,, \tag{1}$$

where $G_{\text{n}} = s_{\text{cr}}\beta(k)\,\dfrac{p_{\text{n}}}{c(T_{\text{n}})}$.

We write the energy equation for sections 1–1 and 2–2 (Fig. 1), and from this equation we determine the thermal resistance, which is the local one, appearing in the region of heat supply.

To exclude from consideration the heat capacities that depend not only on temperature but also on the changing composition of the gas in the zone of combustion, since it complicates the solution of the problem, we assume that in the absence of heat losses the heat content of a fuel is equal to the heat content of combustion products, i.e., $i_{\text{F}} = i_{\text{c.ch}}$ [10]. Then the energy equation of the moving flow of the fuel for the region σ, where it burns out, can be presented as

$$i_1 + \frac{w_1^2}{2} = i_2 + \frac{w_2^2}{2} + \Delta h_{\text{t}}\,. \tag{2}$$

Since $i_1 = i_{\text{F}}$ and $i_2 = i_{\text{c.ch}}$, it follows from Eq. (2) that the thermal resistance is $\Delta t_{\text{t}} = \dfrac{w_1^2}{2} - \dfrac{w_2^2}{2}$ i.e., physically it is the hydraulic resistance, and the magnitude of pressure losses because of heat supply is $h_{\text{t}} = \rho_{\text{c.ch}}\Delta h_{\text{t}}$.

The equation of the balance of momenta for the sections of the flow of a fuel 0–0 and of combustion products 3–3 in the form of $d(mw) = (p_{\text{n}} - h_{\text{fr}}(G) - h_{\text{t}}(G) - h_1(G) - p_{\text{n}})s\,dt$, taking into account that the mass of the combustion is $m = \rho_{\text{c.ch}} ls$, will be written as [9]

$$L_{\text{a,c.ch}} \frac{dG}{dt} = F\,(G) - p_{\text{n}}\,, \tag{3}$$

in which $F(G) = p_{\text{sup}} - h_{\text{fr}}(G) - h_{\text{t}}(G) - h_1(G)$ is the pressure head characteristic of fuel flow in a combustion chamber, with the pressure p_{n} being determined from the relation $G_{\text{n}} = s_{\text{cr}}\beta(k)\,\dfrac{p_{\text{n}}}{c(T_{\text{n}})}$.

When $L_{\text{a,c.ch}} \to 0$, the initial system of equations passes into a degenerate one [2]:

$$C_{\text{a,c.ch}} \frac{dp_{\text{n}}}{dt} = G\,(t-\tau) - s_{\text{cr}}\beta\,(k)\,\frac{p_{\text{n}}}{c\,(T_{\text{n}})}\,, \quad p_{\text{n}} = F\,(G)\,, \tag{4}$$

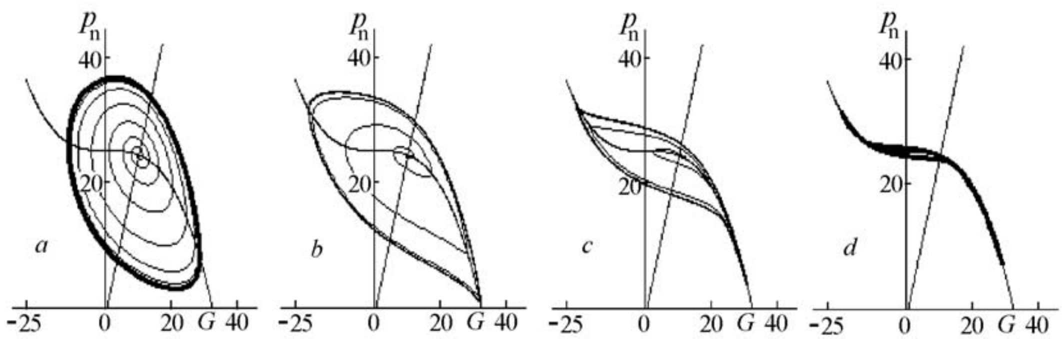

Fig. 2. Variation of the limiting cycles of the system of equations of the LPRE dynamics when this system is transformed into a degenerate form with a monotonically decreasing pressure head characteristic $F(G)$, when p_n = 30 MPa, τ = 0.00017 sec at: a) $L_{a,c.ch}$ = 150 m^{-1}; b) 50; c) 10; d) 0.5.

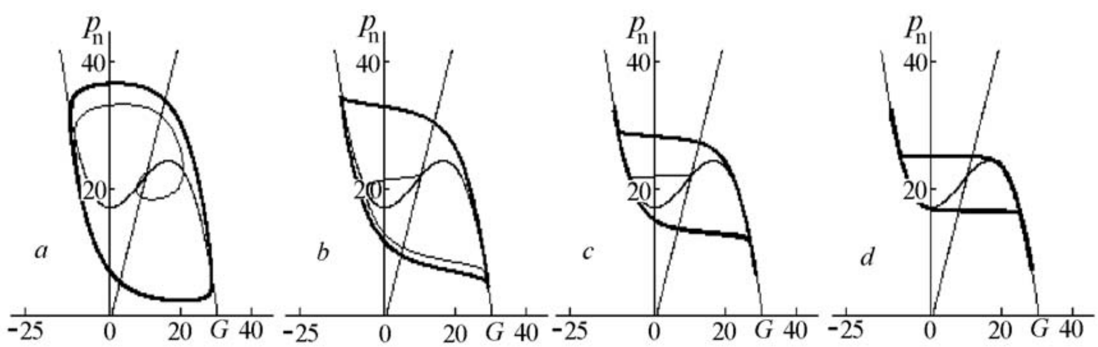

Fig. 3. Variation of the limiting cycles of the system of equations with a saddle-like pressure head characteristic $F(G)$. The values of p_n, τ, and $L_{a,c.ch}$ are same as in Fig. 2.

where $\beta(k) = k\left(\frac{2}{k+1}\right)^{(k+1)/(2(k-1))}$.

Intensity of the Quasi-Elastic Force of Phenomenological Lag τ and Its Influence on the Stability of Flow in the LPRE Combustion Chamber. In the system of equations (1) and (3) we will represent the mass rate of flow of combustion products through a propelling nozzle as $s_{cr}\beta(k)\frac{p_n}{c(T_n)} = \varphi(p_n, T_n)$ and the nonlinear functions $F(G)$ and $\varphi(p_n, T_n)$ as Taylor expansions. For this purpose we express $G = G^* + x$ and $p_n = p_n^* + y$ in terms of new variables, where G^* and p_n^* are the parameters of the stationary regime. Then the considered system of equations (1) and (3) takes the form

$$L_{a,c.ch}\frac{dx}{dt} = F'x + O(x^2) - y\,, \quad C_{a,c.ch}\frac{dy}{dt} = x(t-\tau)\,\varphi' y - O(y^2)\,. \tag{5}$$

The values of G^* and p_n^* are determined by solving the system of equations for the stationary regime of motion of a medium in the LPRE combustion chamber, this system of equations being Eqs. (1) and (3) at $\frac{dG}{dt} = \frac{dp_n}{dt} = 0$.

The characteristic equation of the linear approximation of system (5) has the form

$$C_{a,c.ch}L_{a,c.ch}\lambda^2 - (\varphi' L_{a,c.ch} - C_{a,c.ch}F')\,\lambda - \varphi' F' + \exp(-\lambda\tau) = 0\,. \tag{6}$$

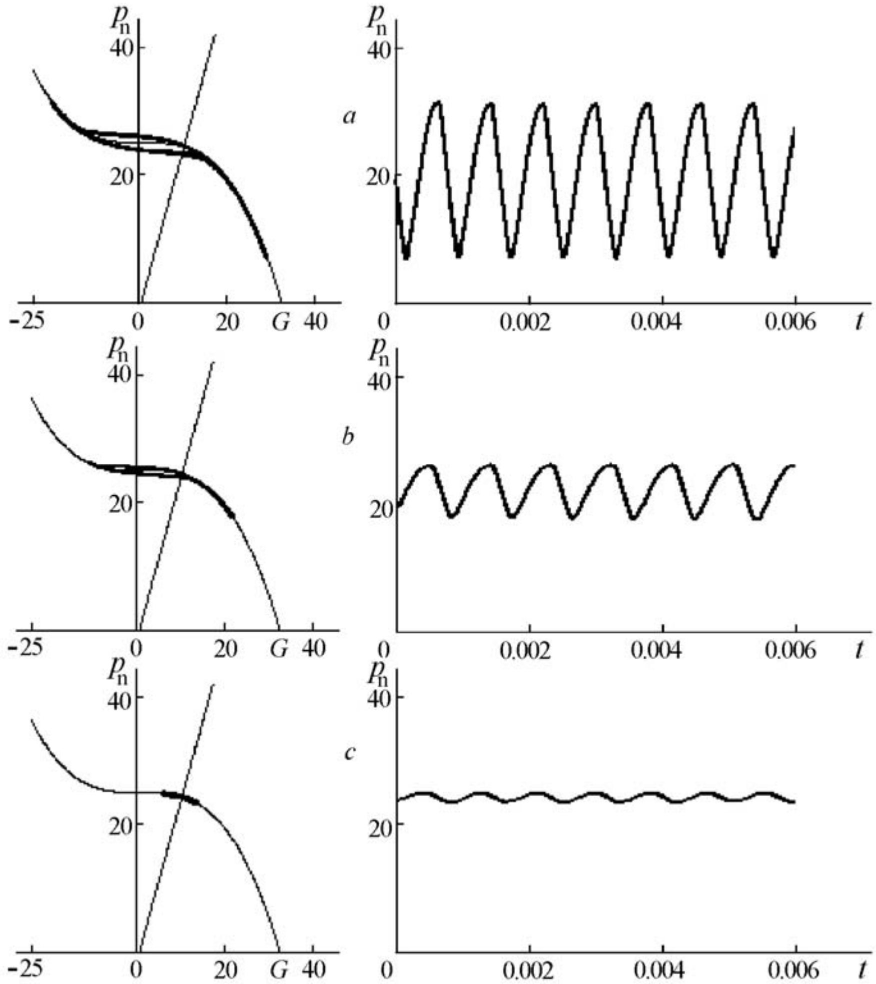

Fig. 4. Decrease in the amplitude of the oscillations of vibration combustion and disappearance of them with an increase in $C_{a,c.ch}$ in the case where the system of equations of fuel motion in an LPRE with a monotonically decreasing pressure head characteristic $F(G)$ is close in form to a degenerate system at: a) $C^*_{a,c.ch} = 2.435 \cdot 10^{-10}$ m·sec^2; b) $C_{a,c.ch} = 2C^*_{a,c.ch}$; c) $C_{a,c.ch} = 2.3C^*_{a,c.ch}$.

Assuming in the characteristic equation (6) that $\lambda = j\omega$, we will obtain two real equations:

$$\varphi' F' - \omega^2 L_{a,c.ch} C_{a,c.ch} - \cos(\omega\tau) = 0\,, \quad \omega\,(\varphi' L_{a,c.ch} - C_{a,c.ch} F') - \sin(\omega\tau) = 0\,, \tag{7}$$

from which we determine the boundaries of the dynamic (vibrational) and static stability [11]. Under the conditions $\sin(\omega\tau) \approx \omega\tau$ and $\cos(\omega\tau) \approx 1$, which can be adopted approximately for low-frequency self-oscillations in an LPRE, the boundary of the region of dynamic stability is determined from the first equation of system (7) as follows:

$$\frac{L_{a,c.ch}}{\xi_n C_{a,c.ch}} - F' - \frac{\tau}{C_{a,c.ch}} \geq 0\,, \tag{8}$$

where $\xi_n = 1/\varphi'$; $\tau/C_{a,c.ch}$ is the intensity of the quasi-elastic force originating because of the phenomenological lag τ; an increase in τ and a decrease in $C_{a,c,ch}$ favors the instability of the stationary regime and the increase in the amplitude of self-oscillations.

The condition of the static instability of the stationary regime of operation of the LPRE combustion chamber $\xi_n - F' \geq 0$ is determined from the second equation of system (7). When it is not satisfied, it follows that the entropy

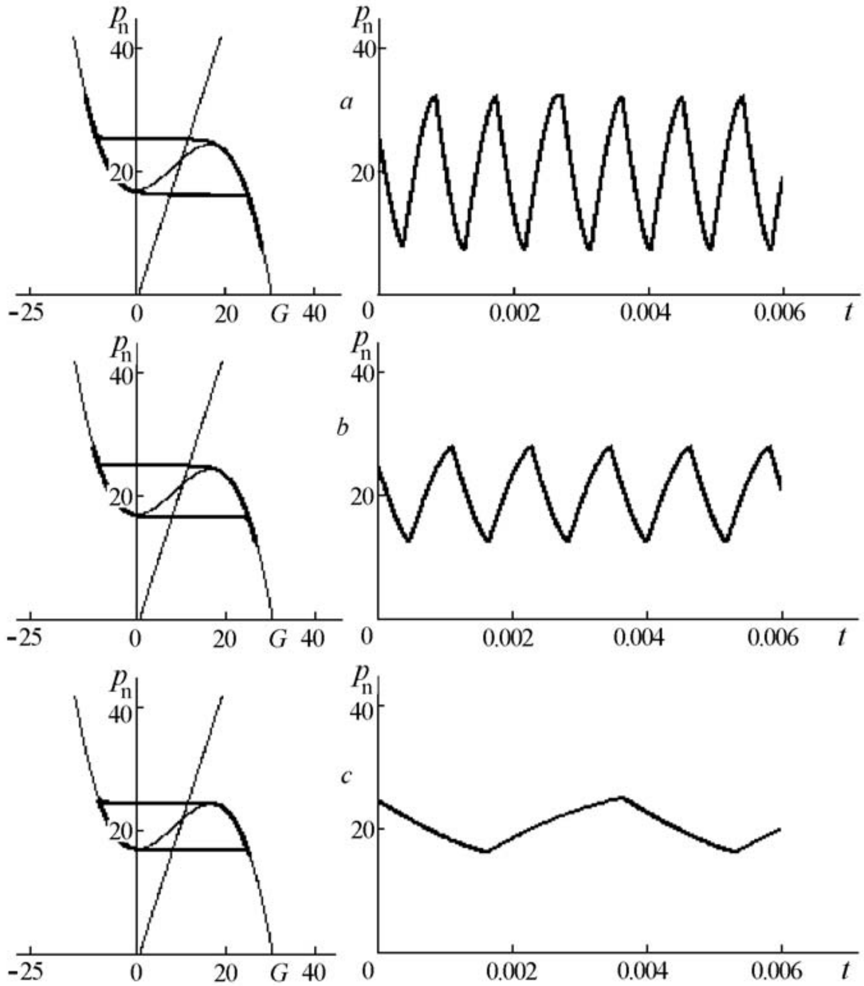

Fig. 5. Transformation of the limiting cycle with an increase in $C_{a,c.ch}$ for the system of equations close to degenerate one with a saddle-like pressure head characteristic $F(G)$ at: a) $C^*_{a,c.ch} = 2.435 \cdot 10^{-10}$ m·sec^2; b) $C_{a,c.ch} = 2C^*_{a,c.ch}$; c) $C_{a,c.ch} = 10C^*_{a,c.ch}$.

waves in the combustion chamber are formed even at a stable monotonically decreasing characteristic $F(G)$ and at $\tau = 0$ because of the presence of the descending branch of the nozzle characteristic in which $\xi_n < 0$ [6].

Limiting Cycles and the Possibility of Controlling Them Using the System of Equations of Motion Transformed into a Degenerate Form. Figures 2 and 3 present the limiting cycles of the system of differential equations (1) and (3). When $L_{a,c.ch} \to 0$, this system passes into the degenerate system (4), when the pressure head characteristic of the combustion chamber is a monotonically decreasing function of the mass flow rate of a fuel (Fig. 2) and has a saddle-like form (Fig. 3).

The decrease in the amplitude of self-oscillations corresponding to the limiting cycle of the system which is close to a degenerate one (Fig. 2d) and their complete suppression can be achieved by increasing the acoustic compliance of the combustion chamber (Fig. 4).

Figure 3d depicts the limiting cycle of the system of equations close to the degenerate system (4), when the pressure head characteristic of the combustion chamber is saddle-like. On increase in the acoustic compliance of the pressure head characteristic branch the constituent parts of this limiting cycle become shorter and disappear. Thereafter, on subsequent increase in the values of $C_{a,c.ch}$ the invirability of the cycle is observed. In this case the amplitudes of oscillations remain constant and only the frequency of relaxation self-oscillations decreases with increase in $C_{a,c.ch}$ (Fig. 5).

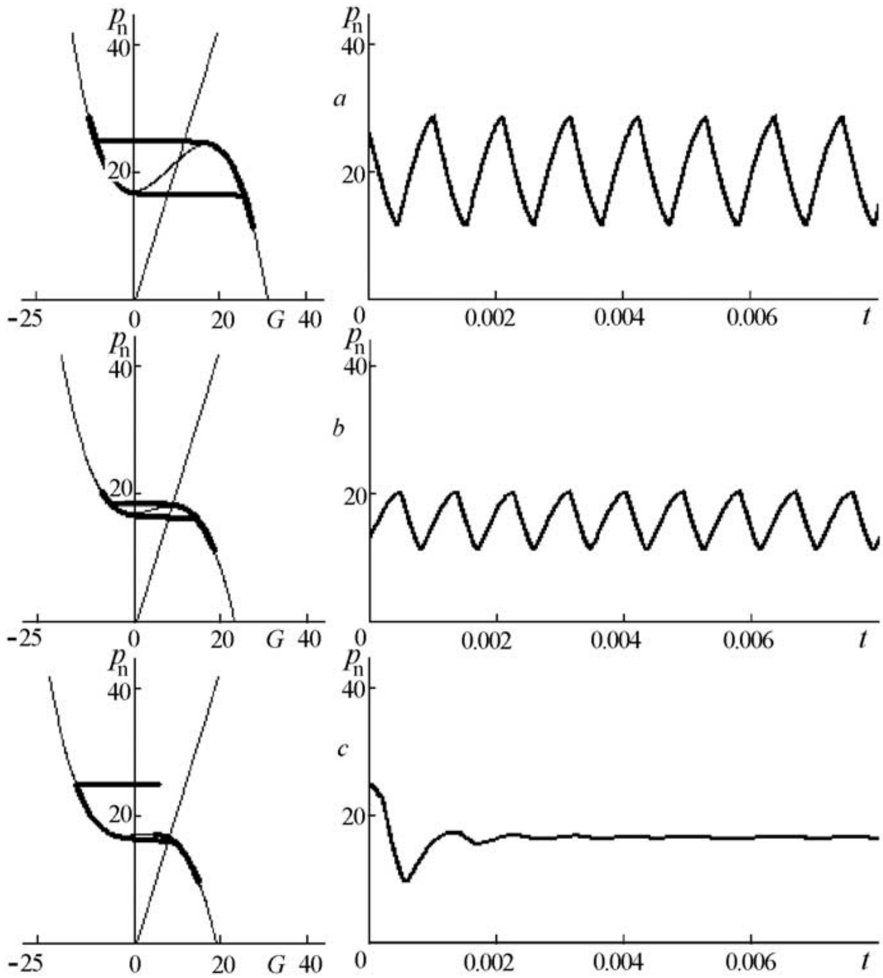

Fig. 6. Variation of the limiting cycle of relaxation self-oscillations by varying the characteristic $F(G)$ by introducing an additional vortex resistance into the combustion chamber, when p_n = 30 MPa, τ = 0.0002 sec, $C_{a,c.ch}$ = 4.871 $\cdot 10^{-10}$ m·sec^2; b) $L_{a,c.ch}$ = 1 m^{-1} at: a) k_{ed} = 0; b) 0.04; c) 0.07.

The suppression of self-oscillations that disappear simultaneously with the destruction of the limiting cycle can be realized by introducing, into the vibrational circuit, the vortex resistance that increases with the fuel flow rate in the combustion chamber, which, in particular, can be attained by increasing the intensity of fuel flow twisting during its spraying [12]. The characteritic of the combustion chamber undergoes a change: $\widetilde{F}(G) = F(G) - k_{ed}G^2$. With increase in the coefficient of eddy losses k_{ed} the value of the amplitude of oscillations decreases and subsequently they disappear entirely (Fig. 6).

The self-oscillations close to harmonic oscillations that correspond to the limiting cycle formed because of the lag τ in burning out, in the absence of the ascending branch of $F(G)$ (Fig. 2a), can be suppressed by increasing the votex resistance [12] without altering the acoustic compliance $C_{a,c,ch}$ of the combustion chamber.

Conclusions. By numerically integrating the system of equations of the nonstationary motion of a medium in an LPRE, in transition of it to a degenerate form, for the first time the possibility of controlling the amplitude of oscillations of vibration combustion at different pressure head characteristic of a combustion chamber has been shown. The possibility of complete suppression of self-oscillations and attainment of a stationary regime of combustion in an LPRE has been established.

NOTATION

$C_{a,c.ch}$, acoustic compliance of combustion chamber, m·sec^2; $c(T_n)$, speed of sound in a flow of combustion products, m/sec; $F(G)$, pressure head characteristic of a fuel flow in a combustion chamber, MPa; G, mass flow rate of a fuel in a combustion chamber, kg/sec; G_n, mass rate of flow of combustion products through the nozzle, kg/sec; G_f, flow rate of a fuel, kg/sec; G_{ox}, flow rate of oxidizer, kg/sec; $h_l(G)$, hydraulic losses along the length, MPa; $\Delta h_t(G)$, specific energy losses by a flow of combustion products because of heat supply, J/kg; $h_{fr}(G)$, hydraulic losses on sprayers, MPa; $h_t(G)$, pressure losses because of heat supply, MPa; $h_n(G, T)$, characteristic of the propelling nozzle, MPa; i_T, heat content of one kilogram of fuel, J/kg; $i_{c.ch}$, heat content of one kilogram of combustion products, J/kg; $j = \sqrt{-1}$, imaginary unit; k, adiabatic index; k_{ed}, coefficient of eddy losses in combustion, MPa/(m^3/sec)^2; l, length, m; $L_{a,c.ch}$, acoustic mass of combustion chamber, m^{-1}; m, mass of a gas, kg; $O(x^2)$, $O(y^2)$, Landau symbols; p_{sup}, pressure in front of sprayers, MPa; p_n, pressure at the inlet into the nozzle, MPa; s, area of normal section of combustion chamber, m^2; s_{cr}, area of critical cross section of the propelling nozzle, m^2; t, time, sec; T, absolute temperature, K; w, velocity of fuel motion after its ignition, m/sec; x, deviation of the mass flow rate of a fuel from its stationary value, m^3/sec; y, deviation of pressure in the combustion chamber before the nozzle inlet from its stationary value, MPa; $\beta(k)$, flow rate dimensionless complex; λ, dimensionless parameter of characteristic polynomials; ξ_n, coefficient of losses of the propelling nozzle, MPa/(m^3·sec)^2; $\rho_{c.ch}$, density of combustion products, kg/m^3; σ, region of fuel combustion, m; τ, value of the lag in combustion, sec; $\varphi(\rho_n, T_n)$, mass rate of flow of combustion products through the nozzle, kg/sec; ω, circular frequency. Subscripts and superscripts: a, acoustic value; c.ch, combustion chamber; cr, critical; ed, eddy; f, fuel; fr, in front of the flame front; F, fuel; len, length; n, nozzle; ox, oxidant; sup, supply; t, refers to temperature; *, parameters of a stationary regime; ~ variaton of pressure characteristic; ′, parameters in section 1–1; 2, parameters in section 2–2; 3, inlet into the propelling nozzle in section 3–3.

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